

Statistics

GFS Summer Prep Packet

Please do your work on a separate sheet of paper. Bring completed work with you to class at the start of the year. Do your best. Know that you will have an opportunity to ask questions if there are problems that you don't know how to do or don't remember fully. There will be a diagnostic assessment in the first few weeks of class, so that your teacher can assess your understanding. The answers are at the end of the document, so check as you go.

Topic: Summation Notation

<http://www.mathsisfun.com/algebra/sigma-notation.html>

Example 1: $\sum_{k=1}^5 3k$

Solution

$$3(1) + 3(2) + 3(3) + 3(4) + 3(5) =$$

$$3 + 6 + 9 + 12 + 15 = 45$$

Example 2: $\sum_{k=1}^4 k^2$

Solution

$$1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$$

Directions: Evaluate each expression. Show work.

A. $\sum_{k=1}^{11} (2k - 7)$

Answer: _____

B. $\sum_{k=1}^9 (8k^2 - 3k)$

Answer: _____

C. $\sum_{k=1}^{10} (4k^2 - 2k + 8)$

Answer: _____

D. $\sum_{k=1}^{100} k$

Answer: _____

Topic: Slope of a Line (given two points)

<http://www.coolmath.com/algebra/08-lines/06-finding-slope-line-given-two-points-01>

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1: Find the slope of the line that passes through (1, 2) and (3, 4).

Solution

$$\frac{2-4}{1-3} = \frac{-2}{-2} = 1$$

Example 2: Find the slope of the line that passes through (3, 6) and (1, 8).

Solution

$$\frac{8-6}{1-3} = \frac{2}{-2} = -1$$

Directions: For the problems below, find the *slope of the line* between each of the two given points. Show work. Write your answer in simplest form.

A. (1, 5) and (7, 8)

Answer: _____

B. (-5, 9) and (5, 11)

Answer: _____

C. (12, 13) and (7, 13)

Answer: _____

D. (-8, 2) and (-8, 7)

Answer: _____

Topic: Permutations

<https://www.mathsisfun.com/combinatorics/combinations-permutations.html>

Permutation Formula (notation): ${}_nP_r = \frac{n!}{(n-r)!}$

Example 1: Evaluate ${}_3P_7$

Solution

$$\frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7(6)(5)(4)(3)(2)(1)}{4(3)(2)(1)} =$$

$$7(6)(5) = \mathbf{210}$$

Example 2: How many different ways can first, second, and third place be awarded to 10 people?

Solution

$n = 10$ people

$r = 3$ (1st, 2nd, 3rd place)

$$\frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10(9)(8)(7)(6)(5)(4)(3)(2)(1)}{7(6)(5)(4)(3)(2)(1)} =$$

$$10(9)(8) = \mathbf{720} \quad 10 \times 9 \times 8 = 720$$

Directions: Evaluate the following expressions. Show work.

A. ${}_{12}P_3$

Answer: _____

B. ${}_8P_5$

Answer: _____

C. In how many ways can a president, a treasurer and a secretary be chosen from among 7 candidates?

Answer: _____

D. How many ways can 1st, 2nd, 3rd, and 4th place be awarded to 10 runners?

Answer: _____

Topic: Combinations

<https://www.mathsisfun.com/combinatorics/combinations-permutations.html>

Combination Formula (notation): ${}_nC_r = \frac{n!}{(n-r)!r!}$

Example 1: Evaluate ${}_3C_7$

Solution

$$\frac{7!}{(7-3)!3!} = \frac{7!}{4!3!} = \frac{7(6)(5)(4!)}{4!(3)(2)(1)} =$$

$$\frac{7(6)(5)}{3(2)(1)} = \frac{210}{6} = 35$$

Example 2: Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose three people to go?

Solution

$$n = 5$$

$$r = 3$$

$$\frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5(4)(3!)}{2(1)(3!)} = \frac{5(4)}{2(1)} = \frac{20}{2} = 10$$

Directions: Evaluate the following expressions. Show work.

A. ${}_{12}C_3$

Answer: _____

B. ${}_8C_5$

Answer: _____

C. In how many ways can three representatives be chosen from among 7 candidates?

Answer: _____

D. Problem: From a group of 40 people, a jury of 12 people is selected. In how many different ways can a jury of 12 people be selected?

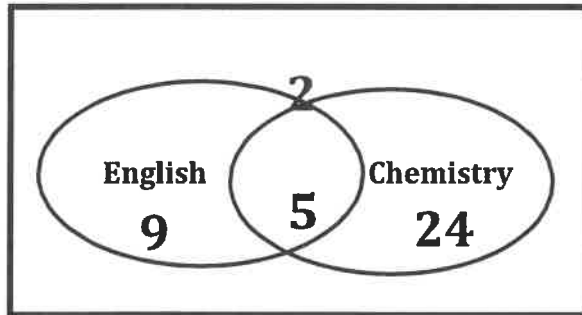
Answer: _____

Topic: Venn Diagrams (2 pages)

<http://www.regentsprep.org/regents/math/algebra/AP2/LVenn.htm>

Example 1: Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, how many students are in neither class?

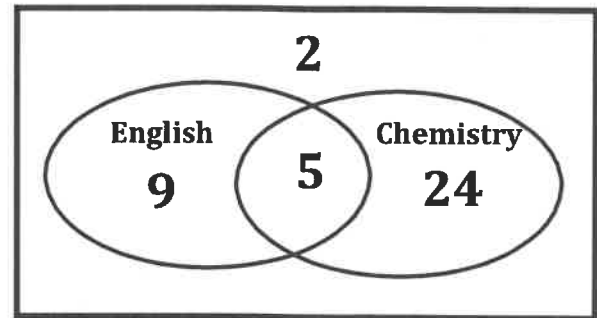
Solution



Two students are taking neither class.

Example 2: Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, how are in either class?

Solution



There are 38 students in at least one of the classes.

Directions: Create a Venn diagram to display the information then solve.

- A. In a class of 87 students, 40 take Chorus, 53 take Band, and 16 take both Chorus and Band. How many students in the class are not enrolled in either Chorus or Band?

B. In a school of 800 students, 185 students are taking Spanish, 160 students are on sports teams, and 72 students participate in both activities. How many students don't participate in either activity?

C. A veterinarian surveys 46 of his patrons. He discovers that 17 have hamsters, 21 have guinea pigs, and 19 have birds. Eight have hamsters and guinea pigs, and 5 people have a guinea pig and a bird. Seven have hamsters and a bird, and of these, 3 people have a guinea pig. How many patrons have none of these pets?

Topic: Writing equations of lines (using slope & y-intercept) – 2 pages

http://www.mathsisfun.com/equation_of_line.html

<http://www.regentsprep.org/regents/math/algebra/AC1/EqLines.htm>

Slope Intercept Form: $y = mx + b$

Example 1: Find the slope of the line that passes through (1, 2) and has a slope of 5.

Solution

Plug in the slope (m) and point (x, y), then solve for b:

$$\begin{aligned}y &= mx + b \\2 &= 5(1) + b \\2 &= 5 + b \\-3 &= b\end{aligned}$$

Now take $m = 5$ and $b = -3$. Plug them in to write your equation in slope-intercept form:

$$y = 5x - 3$$

Example 2: Find the slope of the line that passes through (1, 4) and (3, 10).

Solution

Find the slope first: $\frac{10-4}{3-1} = \frac{6}{2} = 3$

Use the $m = 3$ and any point and plug it in to solve for b:

$$\begin{aligned}y &= mx + b \\4 &= 3(1) + b \\4 &= 3 + b \\1 &= b\end{aligned}$$

Now take $m = 3$ and $b = 1$ and write your equation in slope-intercept form:

$$y = 3x + 1$$

Directions: Write the equation of the line using the given information. Show work.

A. Passes through (2, 4); slope of 3

Answer: _____

B. Passes through (-5, 8); $m = \frac{1}{5}$

Answer: _____

C. Passes through $(0, 0)$; $m = -4$

Answer: _____

D. Passes through $(5, 1)$ and $(3, 0)$

Answer: _____

E. Passes through $(-2, 3)$ and $(-2, -1)$

Answer: _____

Answers

①

$$\begin{aligned} & (2(1)-7) + (2(2)-7) + (2(3)-7) + (2(4)-7) + (2(5)-7) \\ & + (2(6)-7) + (2(7)-7) + (2(8)-7) + (2(9)-7) \\ & + (2(10)-7) + (2(11)-7) \\ & = -5 + (-3) + (-1) + (1) + (3) + (5) + (7) + (9) + (11) + (13) + (15) \\ & = 55 \end{aligned}$$

② $(8(1)^2 - 8(1)) + (8(2)^2 - 8(2)) + (8(3)^2 - 8(3)) + (8(4)^2 - 8(4))$
 $+ (8(5)^2 - 8(5)) + (8(6)^2 - 8(6)) + (8(7)^2 - 8(7))$
 $+ (8(8)^2 - 8(8)) + (8(9)^2 - 8(9))$

$$\begin{aligned} & = (8-8) + (32-8) + (72-24) + (128-32) \\ & + (200-40) + (288-56) + (392-84) \\ & + (512-112) + (648-144) \\ & = 0 + 24 + 48 + 96 + 160 + 232 + 308 + 504 \\ & = 1936 \end{aligned}$$

③ $(4(1)^2 - 2(1) + 8) + (4(2)^2 - 2(2) + 8) + (4(3)^2 - 2(3) + 8)$
 $+ (4(4)^2 - 2(4) + 8) + (4(5)^2 - 2(5) + 8) + (4(6)^2 - 2(6) + 8)$
 $+ (4(7)^2 - 2(7) + 8) + (4(8)^2 - 2(8) + 8)$

$$\begin{aligned} & = (4-2+8) + (16-4+8) + (36-6+8) \\ & + (64-8+8) + (100-10+8) + (144-12+8) \\ & + (196-14+8) + (256-16+8) \\ & = 10 + 20 + 38 + 64 + 98 + 140 + 190 + 248 \end{aligned}$$

$$\textcircled{D} \quad 1 + 2 + 3 + \dots + 100$$

$$= (1+100) + (2+99) + (3+98) + \dots + (50+51)$$

$$= 101 + 101 + 101 + \dots + 101$$

$$= 50(101)$$

$$= 5050$$

$$\textcircled{A} \quad m = \frac{8-5}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$$\textcircled{B} \quad m = \frac{11-9}{5-(-5)} = \frac{2}{10} = \frac{1}{5}$$

$$\textcircled{C} \quad m = \frac{13-13}{7-12} = \frac{0}{-5} = 0$$

$$\textcircled{D} \quad m = \frac{7-2}{-9-(-8)} = \frac{5}{-1} = -5 \Rightarrow \text{undefined.}$$

$$\textcircled{A} \quad {}_{12}P_3 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$

$$= 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 79,833,600$$

$$\textcircled{B} \quad {}_8P_5 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 8 \cdot 7 \cdot 6$$

$$= 336$$

①

$${}^7P_3 = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{\cancel{3 \cdot 2 \cdot 1}} = 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 840$$

②

$${}^{10}P_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}$$

$$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 604,800$$

=

③

$${}^{12}C_3 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(\cancel{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1})(\cancel{3 \cdot 2 \cdot 1})}$$

$$= \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1}$$

$$= \frac{1320}{6}$$

$$= 220$$

④

$${}^8C_5 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(\cancel{3 \cdot 2 \cdot 1})(\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1})}$$

$$= \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$$

$$= \frac{336}{6}$$

$$= 56$$

③

$$7C_3 = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}) (3 \cdot 2 \cdot 1)} = \frac{210}{6} = 35$$

④

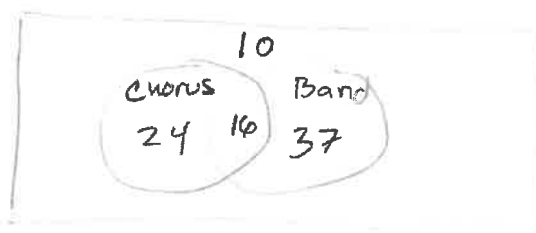
$$40C_{12} = \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35 \cdot 34 \cdot \dots \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(28 \cdot 27 \cdot \dots \cdot 3 \cdot 2 \cdot 1) (12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}$$

$$= \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30 \cdot 29 \cdot 28}{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \frac{7.49 \times 10^{19}}{479001600}$$

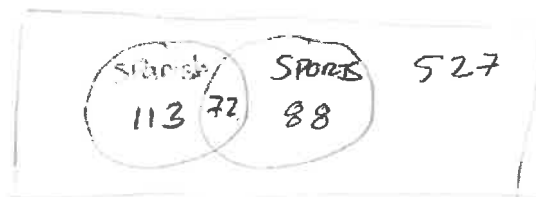
$$= 1.56 \times 10^{11}$$

①



10 Students

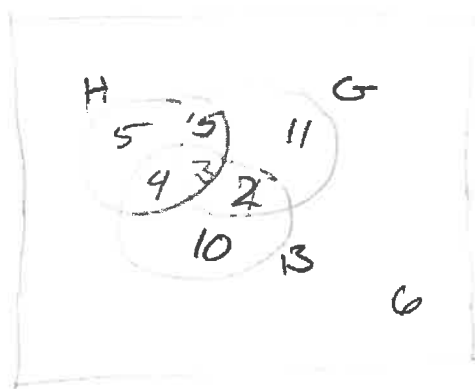
②



527

527 Students

(C)



6 patrons

=

$$(A) \quad y - 4 = 3(x - 2)$$

$$\begin{array}{rcl} y - 4 & = & 3x - 6 \\ +4 & & +4 \end{array}$$

$$y = 3x - 2$$

$$(B) \quad y - 8 = \frac{1}{5}(x + 5)$$

$$\begin{array}{rcl} y - 8 & = & \frac{1}{5}x + 1 \\ +8 & & +8 \end{array}$$

$$y = \frac{1}{5}x + 9$$

$$(C) \quad y - 0 = -4(x - 0)$$

$$y = -4x$$

④

$$m = \frac{0-1}{3-5} = \frac{-1}{-2} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2} (x - 5)$$

$$y - 1 = \frac{1}{2}x - \frac{5}{2}$$

$$\begin{array}{r} +1 \\ \hline y = \frac{1}{2}x - \frac{3}{2} \end{array}$$

⑤

$$m = \frac{-1-3}{-2-(-2)} = \frac{-4}{0} \text{ undefined}$$

$$x = -2$$