

# Calculus

## 2020 Summer Prep Packet

**Please do your work on a separate sheet of paper. Bring completed work with you to class at the start of the year. Do your best. Know that you will have an opportunity to ask questions if there are problems that you don't know how to do or don't remember fully. There will be a diagnostic assessment in the first few weeks of class, so that your teacher can assess your understanding. The answers are at the end of the document, so check as you go.**

1. Copy the table below, and complete it with the proper notation or number line graph.

Inequality	Interval	Graph
$-2 < x \leq 4$		
	$[3, \infty)$	
$\mathbb{R}$ ( <i>all real numbers</i> )		

2. Determine equations of the following lines in **point-slope form**:
  - a. the slope is -4 and the  $y$ -intercept is 5
  - b. the slope is 5 and the line passes through (-6, 71)
  - c. the line that passes through (4, 16) and (-3, 11)
  - d. the line that passes through (-1, 2) and is perpendicular to the line  $2x - 3y + 5 = 0$
  - e. the line that passes through (2, 3) and the midpoint of the segment from (-1, 4) to (3, 2)
3. Find the point of intersection of the lines  $3x - y - 7 = 0$  and  $x + 5y + 3 = 0$  by hand.
4. Solve the following equations for the indicated variables:
  - a.  $A = P + nrP$ , solve for  $P$
  - b.  $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ , solve for  $x$
  - c.  $2x - 2y \frac{dy}{dx} = y + x \frac{dy}{dx}$ , solve for  $\frac{dy}{dx}$
  - d.  $3y^2y' + 2yy' = 5y' + 2x$ , solve for  $y'$
5. For the function  $f(x) = x^2 + 2x + 3$ , find and simplify each of the following:
  - a.  $f(-3)$
  - b.  $f(x + 5)$
  - c.  $f(x + h)$

6. Find the domain of each function:

a.  $f(x) = \frac{3x-1}{x^2-4x-21}$

b.  $g(x) = \sqrt{2x-1}$

c.  $h(x) = \frac{5}{\sqrt{x+3}}$

7. Simplify:

a.  $\log_2 16$

b.  $\log_3 \frac{1}{27}$

c.  $\log_{64} 4$

8. Write the conjugate of each expression.

a.  $3 - 2i$

b.  $4 + \sqrt{7}$

c.  $\sqrt{x+h} - \sqrt{x}$

9. Factor completely:

a.  $2x^2 - 7x + 3$

b.  $x^4 - 1$

c.  $x^6 - 16x^4$

d.  $4x^3 - 8x^2 - 25x + 50$

10. Solve each equation:

a.  $x^2 = -40 + 12x$

b.  $4x^2 + 12x + 3 = 0$

c.  $2x + 1 = \frac{5}{x+2}$

d.  $\frac{x+1}{x} - \frac{x}{x+1} = 0$

11. Describe the end behavior or the following:

$$f(x) = -x^4 + 3x^2 + 3x - 2$$

12. Sketch the general shape of the function:

$$f(x) = 2x^2 - 4x - 4$$

13. Solve for  $x$ .

a.  $|5x - 2| = 8$

b.  $|-x + 4| \leq 1$

14. Use the table to evaluate.

a.  $f(2) =$

b.  $f^{-1}(6) =$

c.  $g(\pi) =$

d.  $g^{-1}(3) =$

$x$	$f(x)$	$g(x)$
2	9	10
$\pi$	6	7
6	4	3

15. Simplify:

a.  $\frac{x^3 - 9x}{x^2 - 7x + 12}$

b.  $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$

16. A water tank has the shape of a cone. The tank is 10m high and has a radius of 3m at the top. If the water in the tank is 5m deep, what is the area of the surface of the water?

17. Two cars start moving from the same point. One travels south at 100 km/h, the other west at 50 km/h. How far apart are they two hours later?

18. A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal?

19. Find the exact value of each:

a.  $\cos 210^\circ$

b.  $\sin \frac{7\pi}{4}$

c.  $\cos \frac{5\pi}{4}$

d.  $\tan \frac{7\pi}{6}$

e.  $\sin^{-1} \frac{\sqrt{2}}{2}$

f.  $\cos^{-1}(-1)$

20. Sketch the graph of each function.

a.  $f(x) = -2x + 5$

b.  $g(x) = \frac{1}{2}x^2$

c.  $h(x) = 5x^3$

d.  $f(x) = \sqrt{x}$

e.  $g(x) = \frac{1}{x}$

f.  $y = e^x$

g.  $g(x) = \sin x$

h.  $h(x) = \cos x$

Answers

1.

Inequality	Interval	Graph
$-2 < x \leq 4$	$(-2, 4]$	Open dot on -2, closed dot on 4, shaded in between
$x \geq 3$	$[3, \infty)$	Closed dot on 3, shaded to the right
$\mathbb{R}$ ( <i>all real numbers</i> )	$(-\infty, \infty)$	Whole number line shaded

2.

a.  $y - 5 = 4(x - 0)$

b.  $y - 71 = 5(x + 6)$

c.  $y - 16 = \frac{5}{7}(x - 4)$

d.  $y - 2 = -\frac{3}{2}(x + 1)$

e.  $y = 3$

3.  $(-1, 2)$

4.

a.  $P = \frac{A}{1+nr}$

b.  $x = \frac{\pi}{\pi-1}$

c.  $\frac{dy}{dx} = \frac{y-2x}{-2y-x}$

d.  $y' = \frac{2x}{3y^2+2y-5}$

5.

a. 6

b.  $x^2 + 12x + 38$

c.  $x^2 + 2xh + h^2 + 2x + 2h + 3$

6.

a.  $x \neq 7, -3$

b.  $x \geq \frac{1}{2}$

c.  $x > -3$

7.

a. 4

b. -3

c.  $\frac{1}{3}$

8.

a.  $3 + 2i$

b.  $4 - \sqrt{7}$

c.  $\sqrt{x+h} + \sqrt{x}$

9.

a.  $(2x - 1)(x - 3)$

b.  $(x - 1)(x + 1)(x^2 + 1)$

c.  $x^4(x - 4)(x + 4)$

d.  $(2x - 5)(2x + 5)(x - 2)$

10.

a.  $\frac{-3 \pm \sqrt{6}}{2}$

b.  $x = \frac{1}{2}, -3$

c.  $x = -\frac{1}{2}$

11. Both ends down

12. Check on calculator

13.

a.  $x = 2 \text{ or } x = -\frac{6}{5}$

b.  $3 \leq x \leq 5$

14.

- a. 9
- b.  $\pi$
- c. 7
- d. 6

15.

- a.  $\frac{x(x+3)}{x-4}$
- b.  $\frac{x-4}{x(x-1)}$

16.  $\frac{9\pi}{4}$

17.  $100\sqrt{5}$

18.  $30^\circ$

19.

- a.  $-\frac{\sqrt{3}}{2}$
- b.  $-\frac{\sqrt{2}}{2}$
- c.  $-\frac{\sqrt{2}}{2}$
- d.  $\frac{\sqrt{3}}{3}$
- e.  $\frac{\pi}{4}$
- f.  $\pi$

20. Check on calculator.