## AP Calculus BC 2020 Summer Prep Packet

Please do your work on a separate sheet of paper. Bring completed work with you to class at the start of the year. Do your best. Know that you will have an opportunity to ask questions if there are problems that you don't know how to do or don't remember fully. There will be a diagnostic assessment in the first few weeks of class, so that your teacher can assess your understanding. The answers are at the end of the document, so check as you go.

1. Simplify 
$$\frac{x-4}{x^2-3x-4}$$
.

2. Simplify 
$$\frac{x^3-8}{x-2}$$
.

3. Simplify 
$$\frac{5-x}{x^2-25}$$
.

4. Simplify 
$$\frac{x^2 - 4x - 32}{x^2 - 16}$$
.

5. Expand 
$$x^{\frac{3}{2}} \left( x + x^{\frac{5}{2}} - x^2 \right)$$

6. Expand and then evaluate the sum 
$$\sum_{n=0}^{5} \frac{(n-1)^2}{2}$$
.

7. Is 
$$f(x) = \sqrt{4 - x^2}$$
 a function? If so, what is its domain?

8. Given 
$$F(x) = \sqrt{x+9}$$
 find and simplify  $\frac{F(x+h) - F(x)}{h}$ ,  $h \neq 0$ .

9. Sketch the graph for 
$$y = \frac{(x+3)(x+2)}{(x-4)(x+1)}$$
.

10. Sketch the graph for 
$$y = \frac{x^2 - 25}{x + 5}$$
.

11. Sketch the graph for 
$$y = \frac{(x^2 - 4)(x - 3)}{x^2 - x - 6}$$
.

12. Sketch the graph for 
$$y = \begin{cases} 9 - x^2 & \text{,} & x \neq -3 \\ 10 & \text{,} & x = -3 \end{cases}$$
.

13. Sketch the graph for 
$$y = \lfloor x \rfloor$$
 ( $\lfloor x \rfloor$  is the greatest integer less than or equal to x)

14. Sketch the graph for  $y = 3 \lfloor 2x \rfloor$ .

15. Sketch the graph for 
$$\begin{cases} x+3 & , & x<-5\\ \sqrt{25-x^2} & , & -5 \le x \le 5\\ 3-x & , & x>5 \end{cases}$$

16. Sketch the graph for y = |x-4|

17. Let 
$$U(x) = \begin{cases} 0 & , & x < 0 \\ 1 & , & x \ge 0 \end{cases}$$

Sketch the graph of:

a. 
$$U(x)$$

b. 
$$xU(x)$$

c. 
$$(x+1)U(x+1)-xU(x)$$

18. Let 
$$sgn(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

Sketch the graph of:

a. 
$$x \operatorname{sgn}(x)$$

b. 
$$x - 2\operatorname{sgn}(x)$$

19. Write  $f(x) = |x^2 - 1|$  as a piecewise function and sketch a graph of that function.

20. Let 
$$f(x) = \frac{x+1}{x-1}$$
 and  $g(x) = \frac{1}{x}$ 

Identify these functions and give their domains:

a. 
$$(f \cdot g)(x)$$

b. 
$$f \circ g(x)$$

c. 
$$f \circ f(x)$$

21. Let 
$$f(x) = \sqrt{x-2}$$
 and  $g(x) = x^2 - 2$ 

Identify these functions and give their domains.

a. 
$$f \circ g(x)$$

b. 
$$g \circ f(x)$$

c. 
$$f \circ f(x)$$

22. Express  $h(x) = \sqrt{x^2 - 4}$  as the composition of two functions.

23. Is 
$$f(x) = \sqrt[3]{x}$$
 even, odd, or neither?

24. Is 
$$f(x) = \frac{x^2 - 5}{2x^3 + x}$$
 even, odd, or neither?

25. Is 
$$f(x) = |x-2| - |x+2|$$
 even, odd, or neither?

26. If 
$$f(x) = x^2, x \le 0$$
 and  $g(x) = -\sqrt{x}$  show that  $f$  and  $g$  are inverse functions.

27. If 
$$f(x) = \{(3,5), (2,4), (1,7)\}$$
,  $g(x) = \sqrt{x-3}$ ,  $h(x) = \{(3,2), (4,3), (1,6)\}$ , and  $k(x) = x^2 + 5$ 

Determine:

a. 
$$(f+h)(1)$$

b. 
$$(k-g)(5)$$

c. 
$$(f \circ h)(3)$$

d. 
$$(g \circ k)(7)$$

e. 
$$f^{-1}(x)$$

f. 
$$g^{-1}(x)$$

g. 
$$\frac{1}{f(x)}$$

28. Rewrite |a| < b without using absolute value bars.

29. Rewrite |a| > b without using absolute value bars.

30. Solve 
$$|2x-3| < 5$$

31. Solve 
$$|3x-2| > 5$$

- 32. The period of (time of one complete oscillation) of a pendulum is directly proportional to the square root of the length of the pendulum. A pendulum of length 8 ft has a period of 2 seconds. Find a mathematical model expressing the period as a function of the length and find the number of swings per second mad by a pendulum of 2 ft in length.
- 33. The surface area of a sphere is given by  $A(r) = 4\pi r^2$ . Suppose a balloon maintains the shape of a sphere as it is being inflated so that the radius is changing at a constant rate of 3 cm per second. If f(t) centimeters is the radius of the balloon after t seconds:
  - a. Compute  $(A \circ f)(t)$  and interpret the result.
  - b. Find the surface area of the balloon after 4 seconds.
- 34. A rectangular field is to be enclosed with 240m of fence but one side of the rectangle is a river so the fencing only needs to be used on the other three sides. Express the area of the field as a function of the length (the dimension parallel to the river), graph the function and give, to the nearest tenth of a meter, the dimensions of the field having the greatest area.
- 35. A manufacturer makes open tin boxes from pieces of tin that are 12 cm by 15 cm by cutting squares out of the corners and bending up the sides. Find the size, to the nearest .01 cm, of the cut-out squares in order that the volume of the boxes is as great as possible.
- 36. Simplify  $\frac{\sqrt{x}}{x}$
- 37. Simplify  $e^{\ln 3}$
- 38. Simplify  $e^{(1+\ln x)}$
- 39. Simplify ln1
- 40. Simplify  $\ln e^7$
- 41. Simplify  $\log_3\left(\frac{1}{3}\right)$
- 42. Simplify  $log_{1/2} 8$
- 43. Simplify  $\ln\left(\frac{1}{2}\right)$

- 44. Simplify  $e^{3 \ln x}$
- 45. Simplify  $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$
- 46. Simplify  $27^{\frac{2}{3}}$
- 47. Simplify  $\left(5a^{\frac{2}{3}}\right)\left(4a^{\frac{3}{2}}\right)$
- 48. Simplify  $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$
- 49. Simplify  $\frac{3(n+2)!}{5n!}$
- 50. Write in slope intercept form the line perpendicular to 2x 3y = 7 and passing through (5, 1) by using the POINT-SLOPE form of a line.
- 51. Find the equation of a line (in slope-intercept form) that is tangent to the circle of radius 2, centered at the origin at a point that is in the center of the second quadrant using the POINT-SLOPE form of a line.
- 52. Simplify  $\sin \theta$
- 53. Simplify  $\sin \frac{\pi}{2}$
- 54. Simplify  $\sin \frac{3\pi}{4}$
- 55. Simplify  $\cos \pi$
- 56. Simplify  $\cos \frac{7\pi}{6}$
- 57. Simplify  $\cos \frac{\pi}{3}$

58. Simplify 
$$\tan \frac{7\pi}{4}$$

59. Simplify 
$$\tan \frac{\pi}{6}$$

60. Simplify 
$$\sec \frac{2\pi}{3}$$

62. 
$$\cos \left( \sin^{-1} \frac{1}{2} \right)$$

63. 
$$\sin^{-1} \left( \sin \frac{7\pi}{6} \right)$$

64. Solve 
$$x^2 + 3x - 4 = 14$$

65. Solve 
$$\frac{x^4 - 1}{x^3} = 0$$

66. Solve 
$$(x-5)^2 = 9$$

67. Solve 
$$2x^2 + 5x = 8$$

68. Solve 
$$(x+3)(x-3) > 0$$

69. Solve 
$$x^2 - 2x - 15 \le 0$$

70. Solve 
$$(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$$

71. Solve 
$$(x-2)(x+3)^7(x-14)^{18}(x+11)^{29}(x)^{34} > 0$$

72. Solve 
$$27^{2x} = 9^{x-3}$$

73. Solve 
$$\log x + \log(x - 3) = 1$$

74. Solve 
$$e^{3x} = 5$$

- 75. Solve  $\ln y = 2x 3$
- 76. Complete the identity sin(A + B) =
- 77. Complete the identity cos(A+B) =
- 78. Complete the identity  $\sin 2A =$
- 79. Complete the identity  $\cos 2A =$
- 80. Complete the identity  $\sin\left(\frac{1}{2}A\right) =$
- 81. Complete the identity  $\cos\left(\frac{1}{2}A\right) =$
- 82. Complete the identity  $\sec^2 A =$
- 83. Complete the identity  $\csc^2 A =$
- 84. In what quadrant is the terminal side of a 100 radian angle that is in standard position?
- 85. Expand  $(a+b)^{8}$
- 86. Find the first four terms of the expansion for  $(2x^2 + y^2)^{12}$ .
- 87. Find the sixth term of the expansion for  $(2x-3)^9$ .
- 88. Find the coefficient of  $x^6$  of the expansion for  $\left[x^2 \left(\frac{1}{x}\right)\right]^{12}$ .
- 89. Find the constant term of the expansion for  $(x^2-2x^{-2})^{10}$
- 90. Find the first five terms for the geometric sequence in which a = -81 and  $r = \frac{1}{3}$ .
- 91. Find the common ration of a geometric series whose third term is -2 and whose sixth term is 54.
- 92. Find the sum of the infinite series 60-6+0.6+...

- 93. Find the sum of the infinite series  $3 + \sqrt{3} + 1...$
- 94. Write the rational number  $1.234234\overline{234}$ ... as a fraction in lowest terms.
- 95. Give the rectangular coordinate for the polar coordinate  $\left(1, -\frac{\pi}{4}\right)$ .
- 96. Give the rectangular coordinate for the polar coordinate  $\left(3, \frac{5\pi}{6}\right)$ .
- 97. Give the rectangular coordinate for the polar coordinate  $\left(-3,\frac{5\pi}{6}\right)$
- 98. Give the rectangular coordinate for the polar coordinate  $\left(-2,-\frac{\pi}{2}\right)$
- 99. Sketch  $r = 2 4\cos\theta$

100. Sketch 
$$r = 3 + 3\sin\theta$$

101. Sketch 
$$r = 2\cos 2\theta$$

Sketch 
$$x = 3 - 2t,$$
$$y = 4 + t$$

103. Sketch 
$$x = 2t^3,$$
$$y = 4t^2$$

Sketch 
$$x = 9\cos t,$$

$$y = 4\sin t, t \in [0,2\pi]$$

- 105. Consider f(x) = 3x 4. What is the greatest distance that x can be from 5 in order that f(x) is no farther than 0.06 away from 11?
- 106. Consider  $f(x) = x^2 + 4$ . What is the greatest distance between x and 2 in order that f(x) is no farther than 0.1 and 8 as one considers values of x moving away from x = 2?
- 107. Draw the complete unit circle from memory.

## Solutions

$$\frac{x-9}{x^2-3x-4} = \frac{x-9}{(x-4)(x+1)} = \frac{1}{x+1}; x \neq 4, -1$$

$$\frac{x^3-8}{x-z} = \frac{(x-z)(x^2+2x+4)}{x-z} = x^2+2x+4; y \neq 2$$

$$\frac{5-x}{x^2-z^5} = \frac{5-x}{x-5(x+5)}$$

$$= \frac{-(x-5)}{x+5} = \frac{-}{x+5}; \quad x \neq \pm 5$$

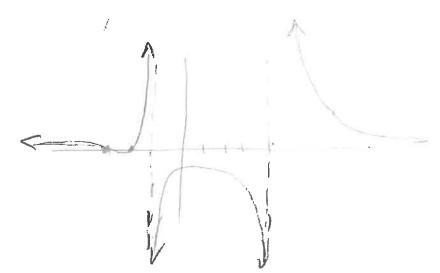
$$9 \frac{x^{2}-4x-32}{x^{2}-16} = \frac{(x-8)(x+4)}{(x-4)(x+4)} = \frac{x-8}{x-4}, \ y \neq \pm 4$$

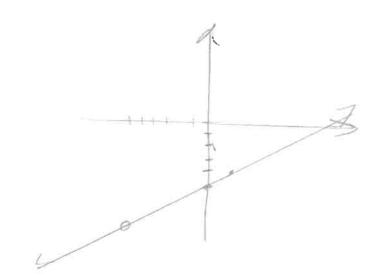
$$\begin{array}{c} 3\frac{3}{2} \times 4 \times 4 \times 5\frac{3}{2} \times 4 \times 5\frac{3}{2} \times 5\frac{$$

$$4-x^2 \ge 0$$

$$-x^2 \ge -4$$

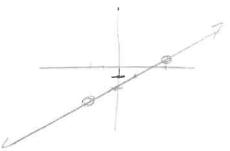


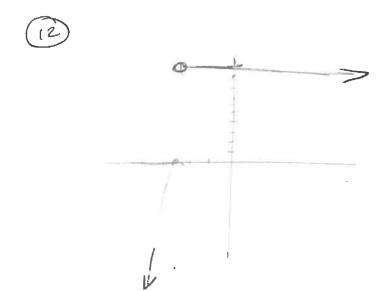


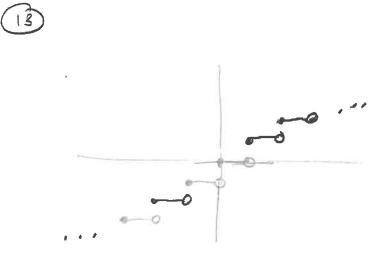


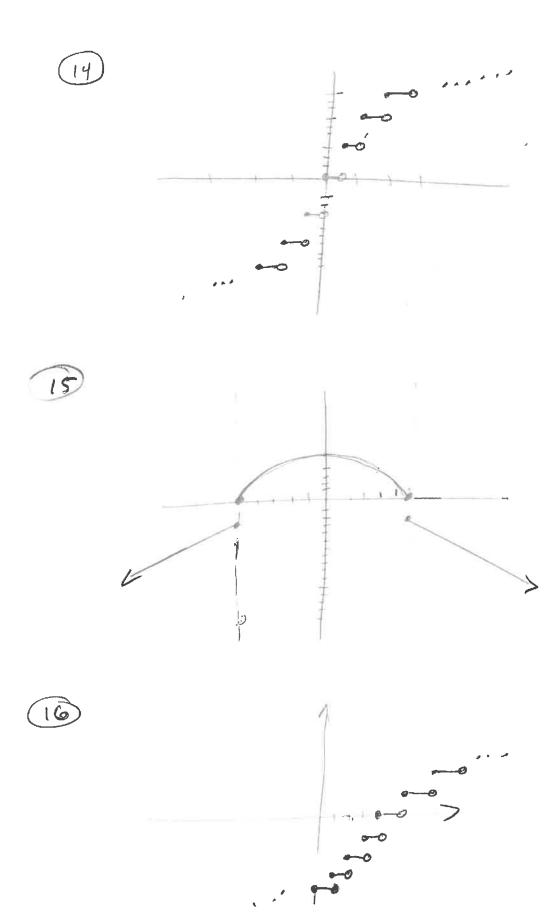
$$\frac{x^2 - 25}{y + 5} = \frac{(x - 5)^2 G_{45}}{x^2}$$

 $(x^{2}-4)(x-3)$   $(x^{2}-4)(x-3)$   $(x^{2}-4)(x-3)$ 

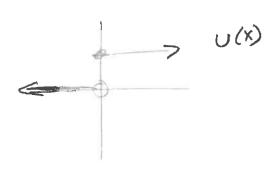


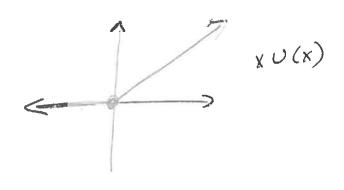






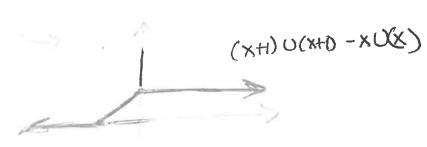




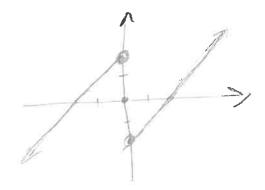


V (X= n(x=)









(19

$$f(x) = \begin{cases} x^{2}-1, & x < -1 \\ -x^{2}+1, & -1 \le x \le 1 \end{cases}$$



$$f(x) = \frac{x+1}{x-1}$$
  $g(x) = \frac{1}{x}$ 

$$\frac{1}{\frac{1}{x}+1} = \frac{1+x}{\frac{1-x}{x}} = \frac{1+x}{\frac{1-x}{x}} = \frac{1+x}{\frac{1-x}{x}}$$

$$= \frac{1+x}{x} \cdot \frac{x}{1-x} = \frac{1+x}{1-x}$$

$$= \frac{1+x}{x} \cdot \frac{x}{1-x} = \frac{1+x}{1-x}$$

(c) 
$$(f \circ f) x = \frac{x+1}{x-1} + \frac{x+1+x-1}{x-1} = \frac{2x}{x-1} \cdot \frac{x-1}{z}$$

$$= X \cdot X \cdot X \neq L$$

-4< x<4

21) (3) 
$$f(x) = \sqrt{x-z}$$
  $g(x) = x^2 - 2$   
 $f(x) = \sqrt{x^2-2} - 2$   
 $= \sqrt{x^2-4}$   $f(x) = x^2 - 2$   
 $= \sqrt{x^2-4}$   $f(x) = x^2 - 2$   
 $= \sqrt{x^2-4}$   $f(x) = x^2 - 2$ 

(a) 
$$(30f) = 1x-2 - 2$$
  
=  $x-2, -2$   
=  $x-4', -70$ 

(c) 
$$f \circ f)(x) = \sqrt{x-2} - 2 | x-2>2 x-2>4 x>6$$

$$h(x) = \sqrt{x^2 - 4}$$

$$h(x) = \sqrt{x^2 - 4}$$

$$h(x) = -x^2 - 4$$

$$h(x) = -x^2 - 4$$

$$f(x) = \sqrt[3]{x}$$

$$f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} \quad \text{opd}$$

$$= -f(x)$$

$$\frac{24}{2x^3+x} + (x) = \frac{x^2-5}{2x^3+x} = \frac{x^2-5}{-(2x^3+x)} = \frac{x^2-5}{-(2x^3+x)} = -4(x) \text{ odd}$$

$$f(x) = |x-z| - |x+z|$$

$$f(x) = |-x-z| - |-x+z| = |-(x+z)| - |-(x-z)|$$

$$= |x+z| - |x-z|$$

$$= -(|x-z| - |x+z| = -|x+z| = -|x+z| = -|x+z| = -|x+z|$$

$$f(x) = x^{2}; x \le 0$$

$$g(x) = -\sqrt{x}$$

$$f(x) = -\sqrt{x}$$

$$f(x) = \sqrt{x} = \sqrt{x}$$

$$0 \quad (f + 1) = f(1) + h(1) = 7 + 6 = 13$$

$$0 \quad (x - 1) (5) = 63 + 6 = 13$$

$$\begin{array}{rcl}
\text{(b)} & (K-g)(5) &=& 5^2 + 5 - (\sqrt{5-3}) \\
&=& 25^2 + 5 - \sqrt{2} \\
&=& 30 - \sqrt{2}
\end{array}$$

(a) 
$$(g \circ k)(7) = g(49+5)$$
  
=  $g(54)$   
=  $\sqrt{54-3}$   
=  $\sqrt{51}$ 

$$\begin{array}{cccc}
A & & & & & & \\
X & & & & & & \\
X & & & & & \\
X & & & & & \\
X & & & & & \\
Y & & & & \\
Y & & & & \\
Y & & & & \\
Y & & & & \\
Y & & & & \\
Y & & & & \\
Y & & & & \\
Y & & & & & \\
Y & & & \\
Y & &$$

(3) 
$$\frac{1}{+(x)} = \{(3, \frac{1}{3}); (2, \frac{1}{4}); (1, \frac{1}{3})\}$$

-b4a4b

$$3x-2^{2} < -5^{2} OR 3x-2>5^{2}$$
  
 $3x < -3$   
 $3x > 3$   
 $3x > 3$   
 $3x > 3$ 

$$P = K \sqrt{1}$$

$$2 = K \sqrt{8}$$

$$P = \sqrt{2}$$

$$P = \sqrt{2}$$

$$P = \sqrt{2} = 1$$

$$P = \sqrt{2}$$

$$P = \sqrt{2} = 1$$

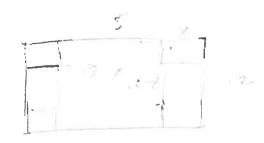
$$P = \sqrt{2}$$

$$P = \sqrt{2} = 1$$

$$A(t) = 4\pi r^2$$
 $A(t) = 3t$ 
 $(A \circ f)(t) = 4\pi (3t)^2$ 
 $= 4\pi (9t^2)$ 
 $= 36\pi e^2$ 
 $(A \circ f)(4) = 36\pi$ 

## 

$$y = 120$$
  $x = 120 - \frac{1}{2}$  (10)  
 $y = 120$   $y = 60$   
 $y = 120$   $y = 60$ 



$$V = (15-x)(12-x)(x)$$

$$= (150 - 27x + x^{2})(x)$$

$$= 180x - 27x^{2} + x^{3}$$

m = 07

$$\frac{\sqrt{y}}{x} = \frac{x^{\frac{1}{2}}}{x} = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$$
 ;  $x \neq 0$ 

$$(49) e^{3\ln x} = e^3$$

$$\frac{4 \times 9^{-2}}{12 \times 3 + 5}$$

$$= \frac{\times (\times^{\frac{1}{3}}) \cdot 9^{5}}{3 \cdot 9^{2}}$$

$$= \frac{\times^{\frac{4}{3}} \cdot 9^{3}}{3 \cdot 3}$$

$$\frac{49}{5(n)!} = \frac{3(n+2)(n+1)}{5}$$

$$2x - 3y = 7$$
 $2x - 7 = 3y$ 
 $y = \frac{2}{3}x - \frac{7}{3}$ 

$$M_{\perp} = -\frac{3}{2} (5.1)$$

$$Sin = \frac{3\pi}{4} IZ$$

$$T = \frac{1}{4} IZ$$



$$= \frac{2\pi}{3}$$

$$= \frac{1}{2}$$

$$= -2$$

Solve 
$$x^2 + 3x - 4 = 14$$
  
 $-14 - 14$   
 $-3x - 28 = 2$   
 $-71(x + 4) = 0$   
 $y = 7$  or  $x = -4$ 

$$2x^{2} = 5 = 8$$
 $2x^{2} = 5 = 8 = 0$ 
 $x = -51/15 = 0$ 
 $x = -5 \pm 1/3$ 

$$(x+ \frac{3}{2} - 9)0$$
 $(x^{2} - 9)0$ 
 $(x^{2} - 9)0$ 

(69) 
$$\chi^{2} - 2\chi - 15 \leq 0$$

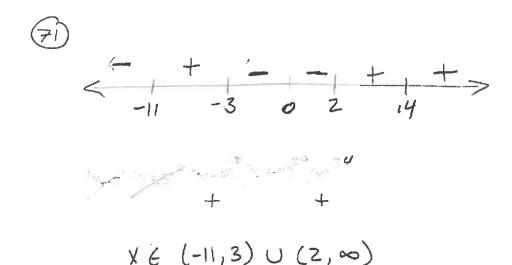
$$(\chi - 5)(\chi + 3) \leq 0$$

$$(x+1)^{2}(x-2) + (x+1)^{2} = 0$$

$$(x+1)^{2}(x-2) + (x+1)^{2} = 0$$

$$(x+1)(x-2)(x+1) = 0$$

$$= -1 \quad x=2 \quad x=\frac{1}{2}$$



$$(3^{3} = 3)$$

$$10^{1} = x(x-3)$$

$$10 = x^2 - 3x$$

$$0 = x^2 - 3x - 0$$

$$X = \frac{\ln 5}{3}$$

$$\begin{array}{cccc}
75 & \ln y = 2x - 3 \\
y = e^{3x}
\end{array}$$

$$\frac{79}{29} \cos 2A = \cos^2 A - \sin^2 A$$
=  $2\cos^2 A - 1$ 
=  $1 - 2\sin^2 A$ 

$$90$$
  $9n(\frac{1}{2}k) = \pm \sqrt{1-100k}$ 

(81) 
$$\cos(\frac{1}{2}A) = \pm \sqrt{\frac{1+405}{2}}$$

$$84$$
 $15.2\pi \approx 94.25$ 
 $100-94.25=5.75$ 
 $\frac{3\pi}{2} \leq 5.75 \leq 2\pi$ 
 $44$  Quad.

= +8a7b+28a +50 3+50 4 +56a3b5+28a3b6+8a57+b8

9 36 94 126 126 74 36 9 L9

Constant term is  $C(x^2)^5(-2x^{-2})^5$   $= C(-2)^5 \frac{x^{10}}{x^{10}}$   $= 252(-2)^5 \frac{x^{10}}{x^{10}}$  = 252(-32) = -8064

-81, -27, -9, -3, -1

-2, -1, -54 -2, -3 = 54  $r^{3} = -27$  r = -3

$$\frac{20}{2} \cos(-\frac{1}{10})^{6-1} = \frac{60}{1-(\frac{1}{10})} = \frac{60}{10}$$

$$= 60 \cdot \frac{10}{10} = \frac{600}{10}$$

$$3 + \sqrt{3} + 1$$

$$\frac{20}{2} 3(\frac{1}{\sqrt{3}})^{2-1} = \frac{3}{1-\frac{1}{63}} = \frac{3}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \left( \sqrt{3} + 1 \right)$$

$$94$$
  $1+\sum_{i=1}^{\infty}.234(\frac{1}{1000})^{i-1}=1+\frac{.234}{1-\frac{1}{1000}}$ 

$$= 1 + \frac{234}{999} = \frac{1233}{999} = \frac{137}{11}$$

$$x = 1 \cos\left(-\frac{\pi}{4}\right)$$

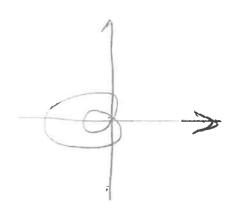
$$X = 1 = 2$$

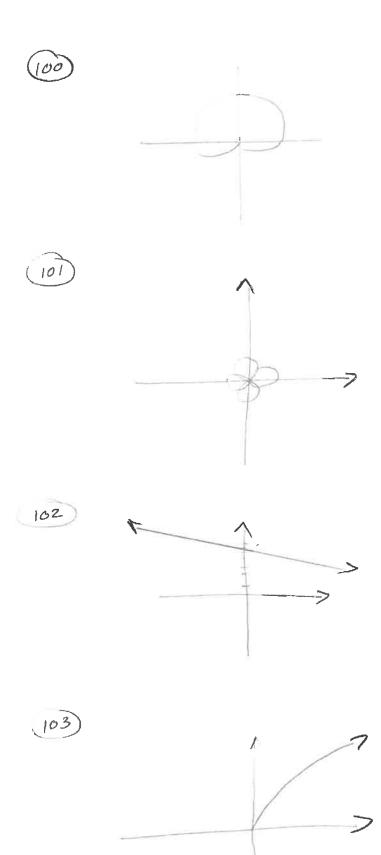
$$\frac{y}{2} = \sqrt{\frac{\pi}{4}}$$

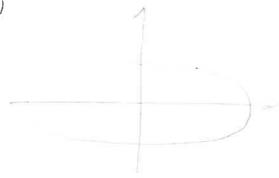
$$= -\frac{\sqrt{2}}{2}$$



$$X = -2 \cos(-\frac{\pi}{2}) \qquad A = -2 \sin(-\frac{\pi}{2})$$







0,2

$$|(x^{2}+4)-8| < 0.1$$

$$|.975< \times < 2.025$$

$$-2 -2 -2$$

$$-0.025< \times -2 < 0.025$$

$$|X-2| < 0.025$$

$$0.025$$

